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Membrane Vacuum as a Type II Superconductor [1]

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Abstract

We study a functional field theory of membranes coupled to a rank–three tensor gauge potential. We show that gauge field radiative corrections lead to membrane condensation which turns the gauge field into a *massive spin–0 field*. This is the Coleman–Weinberg mechanism for *membranes*. An analogy is also drawn with a type–II superconductor. The ground state of the system consists of a two–phase medium in which the superconducting background condensate is "pierced" by four dimensional domains, or "bags", of non superconducting vacuum. Bags are bounded by membranes whose physical thickness is of the order of the inverse mass acquired by the gauge field.

1 Introduction

Of the many aspects of field theory explored by Umezawa during his lifelong research activity, none seems more central and more far reaching than the

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notion of "boson condensation" as a tool to induce structure in the ground state of a physical system. *Boson condensation may lead to the formation of extended objects* [1]. This idea permeates Umezawa's work in the last twenty five years and has inspired numerous original applications in such diverse fields as condensed matter physics, gauge models of particle physics and biology [2].

In retrospect, recognizing the influence of Umezawa's ideas on our own work, we have decided to investigate some new aspects of our current research on the theory of extended objects against the conceptual backdrop of the boson condensation approach. Even though our discussion is applicable to a generic p–brane embedded in a spacetime of arbitrary dimensions, the specific objects that we wish to consider presently are *relativistic bubbles* (2–branes in current terminology), because of their historic role in the development of QCD via the formulation of the so called "bag models" of hadrons and because of their increasingly important role in modern cosmology. In either case, one has to deal with a multiphase ground state characterized by the formation of domain walls separating regions of spacetime with different values of the vacuum energy density. Then, the question that we address in this paper is the search of a mechanism capable of inducing such a structure over the spacetime continuum. One possible answer, we contend, involves the process of boson condensation, and we are fairly confident that Umezawa would agree. We are not equally confident, however, that he would endorse our overall strategy without some qualifications. In fact, before plunging into a technical discussion of our work, it seems appropriate to recall the key conceptual steps of Umezawa's work for the sake of comparison with our own approach.

From Umezawa's vantage point, spatially extended objects, relativistic or not, arise as special solutions of *local quantum field theories* through the process of boson condensation. Some such solutions may have topological singularities, in the sense that the curl of the gradient of the boson condensation function is not necessarily zero. Once formed, extended objects may influence the original quantum system. This " back reaction" may be accounted for by a self–consistent potential attributed to the extended object. The physical paradigm which reflects in full the above logical sequence is a type–II superconductor. In this system, an external magnetic field is squeezed into thin flux tubes by the vacuum pressure of the Cooper pairs condensate. It is this picture that we wish to extend to the case of relativis-

tic bubbles minimally coupled to an antisymmetric tensor gauge potential $A_{\mu\nu\rho}(x)$. More specifically, the purpose of this paper is twofold: first, we wish to show how membrane condensation takes place inducing a two–phase structure in spacetime; second, we wish to show that membrane condensation can be driven by the quantum corrections of the gauge field $A_{\mu\nu\rho}(x)$, in analogy to the Coleman–Weinberg mechanism [3].
All of the above leads us to the interesting technical part of our discussion, to the analogy with superconductivity and to a comparison with Umezawa's approach.

2 The formalism

The picture of *membrane superconductivity*, as opposed to *vortex superconductivity*, can be visualized as "islands" of normal vacuum surrounded by a "sea" of massive $A_{\mu\nu\rho}(x)$–quanta. In general, the emergence of different vacuum phases in the ground state of a physical system is accompanied by the formation of boundary layers between the various vacuum domains. In our approach, these boundaries are approximated by *geometrical manifolds* of various dimensionality (p–branes). This is the point where we depart from Umezawa's approach: p–branes are introduced at the outset with their own action functional, and therefore possess their own dynamics independently of an underlying *local* field theory. *Classical* bubble–dynamics has been studied in detail [4], and this paper represents a tentative step toward the quantum formulation. The paradigm of the quantum approach is a *line field theory* introduced several years ago by Marshall and Ramond as a basis for a second quantized formulation of closed string electrodynamics [5]. We are interested in the case of *relativistic, spatially closed membranes* whose history in spacetime is represented by infinitely thin (1+2)–dimensional Lorentzian submanifolds of Minkowski space (M). In a first quantized approach, membrane coordinates and momenta become operators acting over an appropriate space of states. However, the non linearity of the theory and the invariance under reparametrizations introduce severe problems in the first quantized formulation, e.g. operator anomalies in the algebra of constraints. At least for closed membranes, one can bypass these difficulties by considering a *field theory of geometric surfaces* [6]. If we consider the abstract space F of all possible bubble configurations, then we are led to consider a field theory of

quantum membranes in which the membrane field is a reparametrization invariant, complex, *functional* of the two–surface S which we assume to be the *only* boundary of the membrane history. Our objective, then, is to introduce and discuss the action which governs the evolution of quantum membranes regarded as 3–dimensional timelike submanifolds of Minkowski space. To this end, our first step is to introduce the *3–volume derivative* $\delta/\delta\sigma^{\mu\nu\rho}(s)$, which extends the notion of "loop derivative" introduced, some years ago, in the framework of the loop formulation of gauge theories [7]. The underlying idea is this: suppose we attach at a given point s of the surface S, an infinitesimal, closed surface δS. This procedure is equivalent to a deformation of the initial shape of S in the neighborhood of s, thereby changing the enclosed volume by an infinitesimal amount δV. Then, we define the volume derivative of $\Psi[S]$ through the relation

$$\delta\Psi[S] \equiv \Psi[S \oplus \delta S] - \Psi[S] = \frac{1}{3!}\oint_{\delta V} \frac{\delta\Psi[S]}{\delta\sigma^{\mu\nu\rho}(s)}\, dx^\mu \wedge dx^\nu \wedge dx^\rho \tag{2.1}$$

in the limit of vanishing δV. This definition is "local" to the extent that it involves a single point on the surface. For the whole S , an averaging procedure is required

$$\langle \ldots \rangle \equiv \left(\oint_S d^2 s\sqrt{\gamma}\right)^{-1} \oint_S d^2 s\sqrt{\gamma}\,(\ldots) \tag{2.2}$$

where, $\gamma = x_{\nu\rho}x^{\nu\rho}$ is the determinant of the metric induced over S by the embedding $y^\mu = x^\mu(s^i)$ and $x^{\nu\rho} = \partial(x^\nu, x^\rho)/\partial(s^1, s^2)$ is the surface tangent bi–vector. The volume derivative is related to the more familiar functional variation $\delta/\delta x^\mu(s)$ by the relation

$$\frac{\delta}{\delta x^\mu(s)} = \frac{1}{2}\, x^{\nu\rho} \frac{\delta}{\delta\sigma^{\mu\nu\rho}(s)}. \tag{2.3}$$

Our second step towards the formulation of the membrane wave equation, is to introduce the concept of *monodromy* for the $\Psi[S]$ field, since this notion is directly linked to the physical interpretation of the membrane field. Our requirement is that $\Psi[S] \equiv A[S]e^{i\Theta[S]}$ be a single valued functional of S, i.e. the phase

$$\Theta[S] \equiv \frac{1}{2}\oint_S dx^\mu \wedge dx^\nu \theta_{\mu\nu}(x) \tag{2.4}$$

can vary only by $2\pi n$, $n = 1, 2, ...$, under transport along a "loop" in surface space. This condition constitutes the basis of the analogy with a type–II superconductor. In order to illustrate the precise meaning of this analogy, it is convenient to interpret the motion of a bubble in the abstract space F in which each point corresponds to a possible bubble configuration. Then, the 3–volume derivative introduced above represents the spacetime image of the generator of *translations* in F–space and "classical motion" in F–space corresponds to a continuous surface deformation in Minkowski space. With this understanding, we define a "line" in F–space as a one–parameter family of "points", i.e., surface configurations $\{S; t\}$ in physical space. Let each surface in the family be represented by the embedding equation $x^\mu = x^\mu(s^1, s^2, t = \text{const.})$, where t is the real parameter labelling in a one–to–one way each surface of the family, so that $x^\mu = x^\mu(s^1, s^2; t)$ represents the embedding of the whole family. However, the same relation can be interpreted as the embedding of a single three–surface whose t =const. sections reproduce each surface of the family. In a similar way, we define a "loop" of surfaces as a one–parameter family of surfaces in which the first and the last are identified. Then, according to our definition of volume derivative as the spacetime image of the translation generator in surface space, we define the circulation of $\Theta[S]$ as the flux of the covariant curl of $\theta_{\mu\nu}(x)$:

$$\begin{aligned} \Delta\Theta[S] &\equiv \frac{1}{3!}\oint_\Gamma dx^\mu \wedge dx^\nu \wedge dx^\rho \frac{\delta\Theta[S]}{\delta\sigma^{\mu\nu\rho}(s)} \\ &= \frac{1}{3!}\oint_\Gamma dx^\mu \wedge dx^\nu \wedge dx^\rho \partial_{[\mu}\theta_{\nu\rho]}(x) \end{aligned} \tag{2.5}$$

where $\Gamma : x^\mu = x^\mu(s^1, s^2; t)$, with $\partial\Gamma = \emptyset$ represents the spacetime image of the integration path in surface space.

Finally, we define a *vortex* line in surface space, as a one–parameter family of surfaces $\{V; t\}$ for which the amplitude of the membrane field vanishes, i.e. $|\Psi[V_t]| = 0$. In order to avoid boundary terms and thus simplify calculations, we assume that the vortex line is closed. In other words, the spacetime image of the vortex line is a compact three surface without boundary that we shall denote by ∂B.

Suppose the test loop of surfaces $\Gamma \equiv \partial\Omega$ surrounds the vortex line ∂B, then the monodromy of $\Psi[S]$ implies the quantization condition :

$$\Delta\Theta[S] = \frac{1}{3!}\oint_\Gamma dx^\mu \wedge dx^\nu \wedge dx^\rho \partial_{[\mu}\theta_{\nu\rho]} = 2\pi n \ , \quad n = 1, 2, \ldots \tag{2.6}$$

If the flux (2.6) is quantized, then θ is a singular function within B. Indeed, using Stokes' theorem, we rewrite (2.6) as

$$\begin{aligned}\frac{1}{4!}\int_{\Omega} dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \partial_{[\,\mu}\partial_\nu \theta_{\rho\sigma]} &= \frac{1}{4!}\int_{B} dx^\mu \wedge dx^\nu \wedge dx^\rho \wedge dx^\sigma \partial_{[\,\mu}\partial_\nu \theta_{\rho\sigma]} \\ &= 2\pi n \neq 0. \end{aligned} \tag{2.7}$$

The "Bag" B is the domain of singularity of the phase 2–form $\theta_{\mu\nu}(x)$ and represents the spacetime image of the "vortex interior".

3 The action

After the preparatory discussion of the previous section, we assign to the field $\Psi[S]$ the following action

$$\begin{aligned} S = -\frac{1}{2\cdot 4!}\int d^4x F^{\lambda\mu\nu\rho}F_{\lambda\mu\nu\rho} + \frac{1}{3!}\oint [DS] \langle |\mathcal{D}\Psi[S]|^2\rangle \\ \mathcal{D}_{\mu\nu\rho}\Psi[S] \equiv \left(\frac{\delta}{\delta\sigma^{\mu\nu\rho}(s)} - igA_{\mu\nu\rho}\right)\Psi[S] \end{aligned} \tag{3.1}$$

where, at present, $\oint [DS]\ldots$ is a formal way to write the functional sum over equivalence classes of closed surfaces with respect to reparametrization invariance, and $F_{\lambda\mu\nu\rho} = \partial_{[\mu}A_{\nu\rho\sigma]}$ is the gauge field strength of the rank–three tensor gauge potential $A_{\mu\nu\rho}(x)$. The shorthand notation used in (3.1) is convenient but hides some essential features of the action functional which are worth discussing at this point. From our vantage point, the key property of the action (3.1) is its invariance under the extended gauge transformation

$$\Psi'[S] = \Psi[S]\exp\left(i\frac{g}{2}\oint_S dy^\mu \wedge dy^\nu \Lambda_{\mu\nu}(y)\right) \tag{3.2}$$

$$A'_{\mu\nu\rho} = A_{\mu\nu\rho} + \partial_{[\,\mu}\Lambda_{\nu\rho]} \tag{3.3}$$

This transformation consists of an " ordinary " gauge term for $A_{\mu\nu\rho}(x)$, which is defined over spacetime, and a non local term for the phase of the membrane functional $\Psi[S]$.

The second term in the action contains a spacetime integral which is not

explicitly shown in the expression (3.1). The reason is that a *free* theory of surfaces is invariant under translations of the *center of mass*

$$x^\mu = \langle x^\mu(s) \rangle \tag{3.4}$$

so that the membrane action functional in (3.1) contains the spacetime four–volume as the corresponding zero–mode contribution. This translational invariance is broken by the coupling to an "external" field $A_{\mu\nu\rho}(x)$, in which case the four dimensional zero–mode integral is no longer trivial [8]. It can be factored out by inserting the " unity operator " [9]

$$\int d^4x\, \delta^{4)}\left[\oint_S d^2s \sqrt{\gamma}\,(x - x(s))\right] \left(\oint_S d^2s \sqrt{\gamma}\right) = 1 \tag{3.5}$$

into the functional integral. Then, we define the sum over surfaces as a sum over all the surfaces with the center of mass in x, and then we integrate over x:

$$\begin{aligned} \oint D[S](\dots) &= \int d^4x \oint D[x^\mu(s)]\delta^{4)}\left[x - \langle x(s)\rangle\right](\dots)\ , \\ &\equiv \int d^4x \oint D[S_x](\dots) \end{aligned} \tag{3.6}$$

All of the above applies to the quantum mechanical formulation of surfaces interpreted as geometric objects. On more physical grounds, membranes represent energy layers characterized by a typical thickness, say $1/\Lambda$, which will be determined later on. To take into account the finite thickness of a physical membrane, the singular delta–function which corresponds to the "thin film approximation" , has to be smeared into a regular function sharply peaked around $\langle x^\mu(s)\rangle$. The simplest representation for such a function is given by a momentum space gaussian

$$\delta^{4)}\left[x - \langle x(s)\rangle\right] \rightarrow \delta^{4)}_\Lambda\left[x - \langle x(s)\rangle\right] \equiv \int \frac{d^4k}{(2\pi)^4} e^{-ik_\mu(x^\mu - \langle x^\mu(s)\rangle) - k^2/2\Lambda}. \tag{3.7}$$

However, as long as we work at a distance scale much larger than the membrane transverse dimension, we can approximate the physical extended object with a geometrical surface. In what follows we shall refer to the regularized

delta–function only when it is strictly necessary. With the above prescriptions, the action (3.1) can be written as the spacetime integral of a lagrangian density

$$\mathcal{S}[\Psi^*, \Psi; A_{\mu\nu\rho}] = \int d^4x \left\{ -\frac{1}{2\cdot 4!} F_{\lambda\mu\nu\rho} F^{\lambda\mu\nu\rho} + \frac{1}{3!} \oint D[S_x] \langle \left| \mathcal{D}\Psi[S] \right|^2 \rangle \right\} \quad (3.8)$$

and the interaction between the membrane field current and the $A_{\mu\nu\rho}$ potential is described by

$$\begin{aligned} \mathcal{S}_{\text{int.}}[S; A_{\mu\nu\rho}] &= \frac{g}{2\cdot 3!} \int d^4x \oint D[S_x] \left[i \langle \Psi^*[S] \overleftrightarrow{\mathcal{D}}_{\mu\nu\rho} \Psi[S] \rangle \right] A^{\mu\nu\rho}(x) \\ &= \frac{g}{2\cdot 3!} \int d^4x \oint D[S_x] \left[\langle i\Psi^*[S] \frac{\overleftrightarrow{\delta}}{\delta\sigma_{\mu\nu\rho}(s)} \Psi[S] + g |\Psi[S]|^2 A^{\mu\nu\rho} \rangle \right] A_{\mu\nu\rho}(x) \\ &\equiv \frac{g}{3!} \int d^4x \, J^{\mu\nu\rho}(x) A_{\mu\nu\rho}(x) \end{aligned} \quad (3.9)$$

where g is the gauge coupling constant of dimension two in energy units. As a classical "charge", it describes the strength of the interaction among volume elements of the world–tube swept in spacetime by the membrane evolution. In our functional field theory g enters as the interaction constant between the membrane field current and the gauge potential. Equation (3.9) exhibits a characteristic *London form* which alerts us about the occurrence of non–trivial vacuum phases. Indeed, the current implicitly defined in the last step in equation (3.9), can be rewritten in the following form

$$\begin{aligned} J^{\mu\nu\rho}(x) &= \oint D[S_x] \langle i\Psi^*[S] \frac{\overleftrightarrow{\delta}}{\delta\sigma_{\mu\nu\rho}(s)} \Psi[S] \rangle + g \oint D[S_x] |\Psi[S]|^2 A^{\mu\nu\rho}(x) \\ &\equiv I^{\mu\nu\rho}(x) + \frac{\varphi^2(x)}{g} A^{\mu\nu\rho}(x) \end{aligned} \quad (3.10)$$

where we have introduced the scalar field $\varphi(x)$

$$\varphi^2(x) \equiv g^2 \oint D[x^\mu(s)] \int \frac{d^4k}{(2\pi)^4} e^{-ik_\mu(x^\mu - \langle x^\mu(s) \rangle) - k^2/2\Lambda} |\Psi[S]|^2 \quad (3.11)$$

which we interpret as the *order parameter* associated with membrane condensation in the same way that the Higgs field is the order parameter associated with the boson condensation of point–like objects. In the ordinary vacuum $\varphi(x) = 0$, i.e. there are no centers of mass, and therefore no membranes. Alternatively, we define a vacuum characterized by a constant "density of centers of mass", $\varphi(x) = \text{const.} \neq 0$, as a *membrane condensate*.
We will show in Section 4 that the membrane condensate acts as a *superconductor* upon the gauge potential, turning $A_{\mu\nu\rho}(x)$ into a massive scalar field. The problem of surface condensation is thus reduced to studying the distribution of their representative, pointlike, centers of mass. Conversely, we show in Section 5 that membrane condensation can be driven by the $A_{\mu\nu\rho}(x)$–field quantum corrections alone, and is accounted for by an effective potential ascribed to the extended object. This is Umezawa's self–consistency condition transplanted in our own formalism.

4 Dynamics of the membrane vacuum and the formation of bags

The action (3.1) leads to the pair of coupled field equations

$$\langle \Big| \left(\frac{\delta}{\delta\sigma^{\mu\nu\rho}(s)} - igA_{\mu\nu\rho} \right) \Big|^2 \Psi[S] \rangle = 0 \tag{4.1}$$

$$\partial_\lambda F^{\lambda\mu\nu\rho}(x) = gJ^{\mu\nu\rho}(x) \tag{4.2}$$

which describe the interaction between the membrane field and the $A_{\mu\nu\rho}$ gauge potential. Now, we wish to show that the superconducting membrane condensate contains regions of spacetime, or *bags* of non superconducting vacuum.
Recall that in a type–II superconductor the magnetic field is confined by the superconducting vacuum pressure within a string–like flux tube. Similarly, the membrane condensate confines the gauge field strength within a membrane–like boundary layer surrounding a region of ordinary vacuum. Indeed, in analogy with the superconducting solution of scalar QED, we assume

the following asymptotic boundary condition for $\Psi[S]$

$$\Psi[S] \equiv \frac{\phi}{g^2} e^{i\Theta[S]} = \frac{\phi}{g^2} \exp\left(\frac{i}{2} \oint_S dx^\mu \wedge dx^\nu \theta_{\mu\nu}(x) \right) \tag{4.3}$$

where ϕ is a constant. This is the form of the membrane field when the three volume enclosed by S is much larger than the three volume of the vortex spacetime image. Then, from equation (4.1) we obtain the corresponding asymptotic form of $A_{\mu\nu\rho}$:

$$\left(\frac{\delta}{\delta\sigma^{\mu\nu\rho}(s)} - gA_{\mu\nu\rho} \right) \Psi[S] = 0 \quad \longrightarrow A_{\mu\nu\rho} = \frac{1}{g} \partial_{[\mu} \theta_{\mu\nu]} . \tag{4.4}$$

Therefore, the flux of $F_{\lambda\mu\nu\rho}$ across a large four dimensional region Ω enclosing B is given by

$$\begin{aligned} q_n &\equiv \frac{1}{4!} \int_\Omega F_{\lambda\mu\nu\rho}\, dx^\lambda \wedge dx^\mu \wedge dx^\nu \wedge dx^\rho \\ &= \frac{1}{3!} \oint_\Gamma dx^\mu \wedge dx^\nu \wedge dx^\rho A_{\mu\nu\rho} \\ &= \frac{1}{3! g} \oint_\Gamma dx^\mu \wedge dx^\nu \wedge dx^\rho \partial_{[\,\mu} \theta_{\nu\rho]} = \frac{2\pi n}{g} \qquad n = 1, 2, \ldots \end{aligned} \tag{4.5}$$

Thus, the physical consequence of the monodromy of $\Psi[S]$ is that the flux of $F_{\lambda\mu\nu\rho}$ through a region enclosing B is quantized in units of $2\pi/g$.
In the superconducting phase, equation (4.2) becomes

$$\partial_\lambda F^{\lambda\mu\nu\rho}(x) = -\phi^2 \left(A^{\mu\nu\rho}(x) - \frac{1}{g} \partial^{[\,\mu} \theta^{\nu\rho]}(x) \right) \equiv -j^{\mu\nu\rho}(x) \tag{4.6}$$

in which we have introduced the *supercurrent density* $j^{\mu\nu\rho}(x)$. Equation (4.6) holds only where $\theta^{\nu\rho}(x)$ is a regular function. In the domain of singularity, where the partial derivatives of $\theta^{\nu\rho}(x)$ do not commute, the covariant curl of $\partial^{[\,\mu} \theta^{\nu\rho]}(x)$ should be interpreted in the sense of distribution theory. Indeed, if we apply the covariant curl operator to both sides of equation (4.6), we obtain

$$\partial^{[\,\lambda} j^{\mu\nu\rho]} = -\phi^2 \left(F^{\lambda\mu\nu\rho} - \frac{1}{g} \partial^{[\,\lambda} \partial^\mu \theta^{\nu\rho]} \right) \tag{4.7}$$

The last term in (4.7) may not be disregarded without violating (2.7). Therefore in order to match (4.5) with (2.7), we define

$$\begin{aligned} \partial^{[\lambda}\partial^{\mu}\theta^{\nu\rho]}(x) &\equiv \frac{q_n g}{\Omega_B}\epsilon^{\lambda\mu\nu\rho}\int_B d^4\xi\,\delta^{4)}\left[x - z(\xi)\right] \\ &\equiv \frac{q_n g}{\Omega_B} J_B^{\lambda\mu\nu\rho}(x) \end{aligned} \tag{4.8}$$

where, Ω_B and $J_B^{\lambda\mu\nu\rho}(x)$ are, respectively, the bag four–volume and the bag current. Thus, the supercurrent can be determined from the equation

$$\partial_\lambda \partial^{[\lambda} j^{\mu\nu\rho]} = -\phi^2 \left(j^{\mu\nu\rho} - \frac{q_n}{\Omega_B}\partial_\lambda J_B^{\lambda\mu\nu\rho} \right) \tag{4.9}$$

by means of the Green function method:

$$j^{\mu\nu\rho}(x) = -\frac{\phi^2 q_n}{\Omega_B}\epsilon^{\mu\nu\rho\sigma}\int_B d^4 z\,\partial_\sigma\, G(x - z; \phi^2) \tag{4.10}$$

where $G(x - z)$ is the scalar Green function

$$\left[\partial^2 + \phi^2\right] G(x - z; \phi^2) = \delta^{4)}(x - z). \tag{4.11}$$

Then, from equation (4.2), (4.7) and (4.10), we find the form of the confined gauge field

$$F^{\lambda\mu\nu\rho}(x) = -\frac{\phi^2 q_n}{\Omega_B}\epsilon^{\mu\nu\rho\sigma}\int_B d^4 z\, G(x - z; \phi^2). \tag{4.12}$$

The analogy between the membrane vacuum and a type–II superconductor now seems manifest: in the ordinary vacuum $A_{\mu\nu\rho}$ does not propagate any degree of freedom. Rather, it corresponds to a uniform energy background. However, in the superconducting phase $A_{\mu\nu\rho}$ becomes a dynamical field describing a massive, spin–0 particle [10]. The source for the massive field is the bag current (4.8). In a boson particle condensate, the magnetic field is confined to a thin flux tube surrounding the vortex line; in the membrane condensate, the $F_{\lambda\mu\nu\rho}$–field is confined within the membrane which encloses the ordinary vacuum bag. The gauge field provides the "skin" of the bag. To complete the analogy, in the next section we show that the thickness of the membrane is given by the inverse of the dynamically generated mass of $F_{\lambda\mu\nu\rho}$.

5 The Coleman–Weinberg mechanism of mass generation

The scenario emerging from the last section is based on the assumption that the $\varphi(x)$–field can acquire a non vanishing vacuum expectation value. Note that there is no potential term for $\varphi(x)$ in the classical action (3.1). However, we wish to show that a self–consistent potential may originate from the quantum fluctuations of the $A_{\mu\nu\rho}(x)$ field leading to a non vanishing vacuum expectation value for the order parameter. On the technical side, this means to compute the one–loop effective potential for the $\varphi(x)$ field by integrating $A_{\mu\nu\rho}$ out of the functional integral

$$Z = \int D\left[\Psi^*[S]\right] D\left[\Psi[S]\right]\left[DA_{\mu\nu\rho}(x)\right] \exp\left(i\mathcal{S}/\hbar\right). \tag{5.1}$$

The quantization of higher rank gauge fields is a lengthy procedure involving a sequence of gauge fixing conditions together with various generations of ghosts [11]. In principle, these terms should be included in the functional measure in the action functional. However, in our case they are unnecessary since we know already that in the superconducting phase $A_{\mu\nu\rho}$ describes a massive scalar degree of freedom which is the *only physical degree of freedom*. Thus, the effective potential is

$$\begin{aligned} V^{\rm eff}(\phi) &= \frac{1}{2}\mathrm{Tr}\,\ln\left[\left(\partial^2+\phi^2\right)/\Lambda^2\right] + \frac{\delta\rho(\Lambda)}{2}\phi^2 + \frac{\delta\lambda(\Lambda)}{4!}\phi^4 \\ &= \frac{1}{32\pi^2}\left[\phi^2\Lambda^2 + \frac{\phi^4}{2}\left(\ln\left(\frac{\phi^2}{\Lambda^2}\right)-\frac{1}{2}\right)\right] + \frac{\delta\rho(\Lambda)}{2}\phi^2 + \frac{\delta\lambda(\Lambda)}{4!}\phi^4. \end{aligned}$$

The ultraviolet divergences of the one–loop determinant have been regularized through the cutoff Λ, and the two counterterms $\delta\rho(\Lambda)$ and $\delta\lambda(\Lambda)$ are fixed by the renormalization conditions

$$\left(\frac{\partial^2 V^{\rm eff}(\phi)}{\partial\phi^2}\right)_{\phi=0} = 0 \tag{5.2}$$

$$\left(\frac{\partial^4 V^{\rm eff}(\phi)}{\partial\phi^4}\right)_{\phi=g/\mu} = 0 \tag{5.3}$$

in which μ appears as an arbitrary renormalization scale. The scalar field $\varphi(x)$ has no classical dynamics of its own, i.e., it possesses no kinetic or

potential term. This is the reason for imposing the two conditions (5.2),(5.3): equation (5.2) is the characteristic Coleman–Weinberg condition [3] ensuring that the mass of the gauge field is non vanishing only in the condensed phase $\phi \neq 0$; equation (5.3) follows from the absence of a classical quartic self–interaction. Of course, the physical properties of the system are insensitive to the choice of the renormalization condition. Then, with our choice, we find the *Coleman–Weinberg potential for membranes*

$$V_{CW}(\phi) = \frac{\phi^4}{64\pi^2}\left[\ln\frac{\phi^2\mu^2}{g^2} - \frac{25}{6}\right]. \tag{5.4}$$

The absolute minimum of $V_{CW}(\phi)$ corresponds to a super–conducting phase characterized by a vacuum expectation value of the order parameter

$$\langle\phi\rangle^2 = \frac{g^2}{\mu^2} e^{\frac{11}{3}} \tag{5.5}$$

and by a dynamical surface tension

$$\frac{\rho_R^2}{g^2} \equiv \left(\frac{\partial^2 V_{CW}(\phi)}{\partial\phi^2}\right)_{\phi=\langle\phi\rangle} = \frac{g^2}{8\pi^2\mu^2} e^{\frac{11}{3}}. \tag{5.6}$$

The factor $\langle\phi\rangle^2$ as given by (5.5) is also the square of the dynamically generated mass for $A_{\mu\nu\rho}(x)$. Hence, the physical thickness of the membrane, is of the order of $\langle\phi\rangle^{-1}$, and the dynamically generated surface tension is $\rho_R = g\langle\phi\rangle/\sqrt{8\pi^2}$. This quantity is positive, so that the bubbles of ordinary vacuum tend to collapse in the absence of a balancing internal pressure. In the case of "hadronic bags" , this internal pressure is provided by the quark–gluon complex. In any event, the picture of the superconducting membrane vacuum is strongly reminiscent of the *classical* dynamics of a closed membrane coupled to its gauge partner i.e., $A_{\mu\nu\rho}(x)$ [4]. In both cases, vacuum bubbles created in one vacuum phase evolve and die in a different vacuum background. This suggests a new possibility of quantum vacuum polarization via the creation and annihilation of whole domains of spacetime in which the energy density is different from that of the ambient spacetime. As a matter of fact, the novelty of our field model is the onset of a new type of "Higgs mechanism for membranes" triggered solely by quantum fluctuations. The

effect of such fluctuations can be accounted for by an effective potential. As in Umezawa's approach, this effective potential is consistent with the dynamical generation of a bag with surface tension out of the vacuum.

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